Static Balancing cont.

- Let's consider the unbalanced link shown below. We would like to balance the link with a single mass m_b.

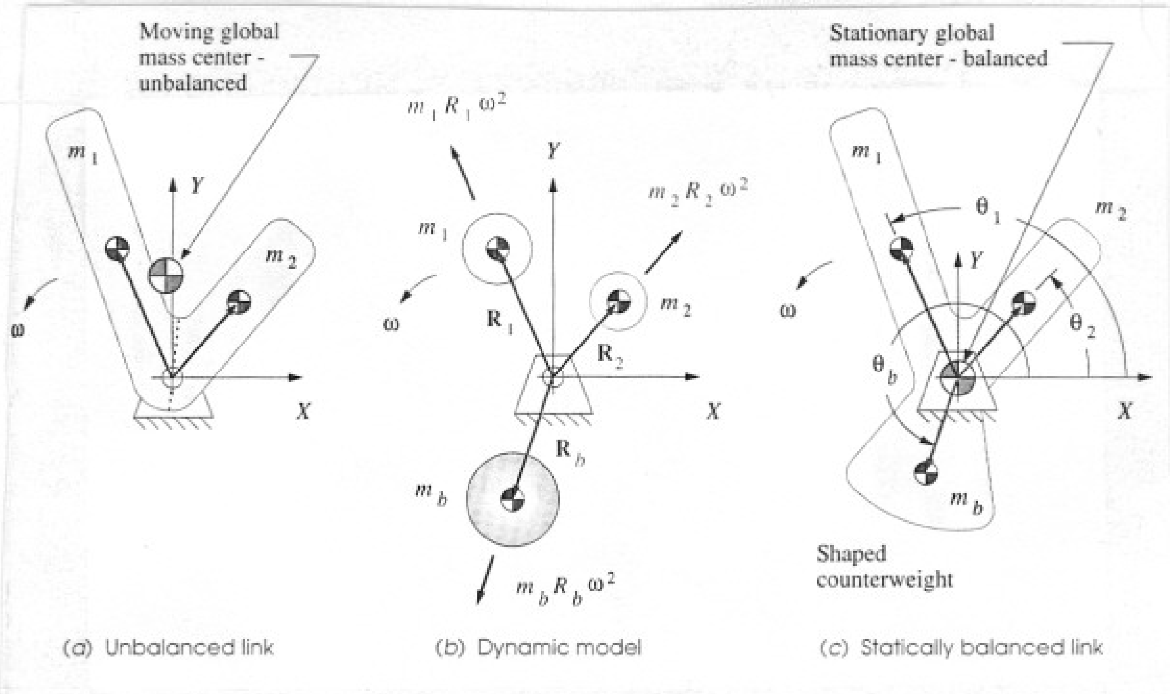

FIGURE 12-1
Static balancing a link in pure rotation

In order to do this we need to essentially make the center of mass of m_1, m_2, and m_b be located at $x = 0$, $y = 0$.

$$\bar{y} = \sum m_i y_i = 0 = m_b R_{by} + m_1 R_{1y} + m_2 R_{2y}$$

$$\bar{x} = \sum m_i x_i = 0 = m_b R_{bx} + m_1 R_{1x} + m_2 R_{2x}$$

we can re-arrange the terms and place the unknowns on the LHS

$$m_b R_{bx} = -(m_1 R_{1x} + m_2 R_{2x})$$

$$m_b R_{by} = -(m_1 R_{1y} + m_2 R_{2y})$$

Now we can divide these equations and take the inverse tangent of both sides to get θ_b

(1) $$\theta_B = \tan^{-1}\left(\frac{m_b R_{by}}{m_b R_{bx}}\right) = \tan^{-1}\left(\frac{m_1 R_{1y} + m_2 R_{2y}}{m_1 R_{1x} + m_2 R_{2x}}\right)$$

$$m_b R_b = m_b \sqrt{R_{bx}^2 + R_{by}^2} = \sqrt{(m_b R_{bx})^2 + (m_b R_{by})^2}$$

We can substitute the two equations on bottom of p. 19-1

(2) $$m_b R_b = \sqrt{(m_1 R_{1x} + m_2 R_{2x})^2 + (m_1 R_{1y} + m_2 R_{2y})^2}$$

Using Equations (1) and (2) we can compute the orientation and the product of the mass x radius required to balance the linkage.

Example

For the system shown on p. 19-1

$m_1 = 1.2\,kg$, $m_2 = 1.8\,kg$, $R_1 = 1.135\,m \angle 113.4°$

$R_2 = 0.822\,m \angle 48.8°$

Find: the mass-radius product and its orientation to statically balance the system

Solution:

$R_{1x} = -0.451\,m$ $R_{1y} = 1.042\,m$

$R_{2x} = 0.541\,m$ $R_{2y} = 0.618\,m$

$$m_1 R_{1x} + m_2 R_{2x} = (1.2)(-0.451) + (1.8)(.541) = 0.433$$

$$m_1 R_{1y} + m_2 R_{2y} = (1.2)(1.042) + (1.8)(0.618) = 2.363$$

Substitute into (1 + 2) →

(1) → $\theta_b = \tan^{-1}\left(\frac{2.363}{0.433}\right) = 79.6°$ Note: this is the orientation of the imbalance center of gravity. In order to find the orientation of the balance weight, we need to add 180° $\therefore$ $\boxed{\theta_b = 259.6°}$

$$m_b R_b = \sqrt{0.433^2 + 2.363^2} = \boxed{2.402 \text{ kgm} = m_b R_b}$$

We can use pretty much any balance weight as long as the mass-radius product is equal to 2.402 kgm.

Dynamic Balancing cont.

Let's consider the two plane dynamic balancing problem

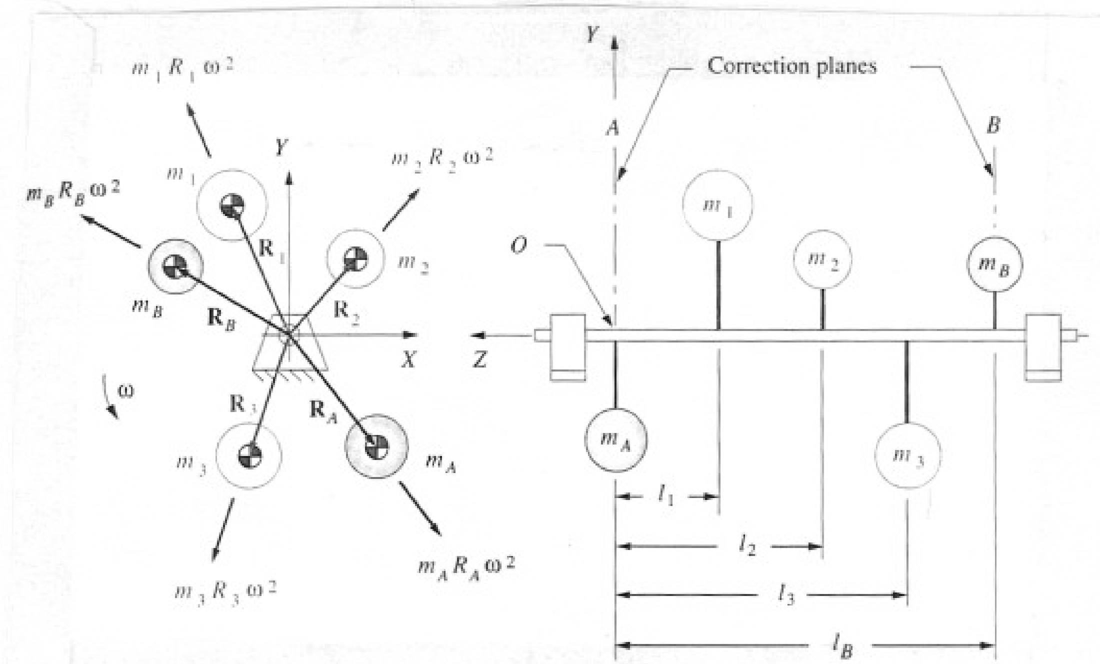

FIGURE 12-3
Two-plane dynamic balancing

m_1, m_2, and m_3 represent the masses that cause the imbalance. m_a and m_b represent the balancing weights that are added to the shaft to balance the system.

If we look at the problem from the side (XY) the problem looks similar to the one we solved on p. 19-1. So let's start by making $\bar{x} = \bar{y} = 0$

$$\bar{x} = \sum m_i x_i = 0 = m_A R_{Ax} + m_B R_{Bx} + m_1 R_{1x} + m_2 R_{2x} + m_3 R_{3x}$$

$$\bar{y} = \sum m_i y_i = 0 = m_A R_{Ay} + m_B R_{By} + m_1 R_{1y} + m_2 R_{2y} + m_3 R_{3y}$$

Rearranging terms

$$m_A R_{Ax} + m_B R_{Bx} = - m_1 R_{1x} - m_2 R_{2x} - m_3 R_{3x}$$

$$\underbrace{m_A R_{Ay} + m_B R_{By}}_{\text{4 unknown terms}} = \underbrace{- m_1 R_{1y} - m_2 R_{2y} - m_3 R_{3y}}_{\text{Known terms}}$$

In order to solve for the unknowns we need two additional equations. We can sum moments in the XZ and YZ planes about point o.

<u>XZ plane (about the Y axis)</u>

$$\sum M = 0 = (m_B R_{Bx})\ell_B + (m_1 R_{1x})\ell_1 + (m_2 R_{2x})\ell_2 + (m_3 R_{3x})\ell_3$$

Rearranging terms

$$m_B R_{Bx} = \frac{-(m_1 R_{1x})\ell_1 - (m_2 R_{2x})\ell_2 - (m_3 R_{3x})\ell_3}{\ell_B}$$

<u>YZ plane (about the X axis)</u> similarly

$$m_B R_{By} = \frac{-(m_1 R_{1y})\ell_1 - (m_2 R_{2y})\ell_2 - (m_3 R_{3y})\ell_3}{\ell_B}$$

To find $m_A R_{Ax}$ and $m_A R_{Ay}$ we can substitute 19-5 into the equations on p. 19-4

$$m_A R_{Ax} = -m_B R_{Bx} - m_1 R_{1x} - m_2 R_{2x} - m_3 R_{3x}$$

$$m_A R_{Ay} = -m_B R_{By} - m_1 R_{1y} - m_2 R_{2y} - m_3 R_{3y}$$

Lastly to find the orientation of the balance weights for correction plane A and B we can use the equation from the top of p. 19-2

$$\theta_B = \tan^{-1}\left(\frac{m_B R_{By}}{m_B R_{Bx}}\right)$$

$$\theta_A = \tan^{-1}\left(\frac{m_A R_{Ay}}{m_A R_{Ax}}\right)$$

Note: The preceding equations can be generalized to any number of imbalance masses $m_1, m_2, m_3, \ldots m_n$ simply by adding extra terms.